Math 101 - Chapter 5/6 Review

#1. $-9a^5(-3a^6 - 2a^4 + 8a^2)$

$= (-9a^5)\cdot(-3a^6) + (-9a^5)(-2a^4) + (-9a^5)(8a^2)$

$= 27a^{11} + 18a^9 - 72a^7$

#2. $-4.6m^2n(1.3m^4n^2 - 2.6m^3n^3 + 5.9n^4)$

$= (-4.6m^2n)(1.3m^4n^2) + (-4.6m^2n)(-2.6m^3n^3) + (-4.6m^2n)(5.9n^4)$

$= -5.98m^6n^3 + 11.96m^5n^4 - 27.14m^2n^5$

#3. $(5x+7)(3y-8)$

$= (5x)(3y) + (5x)(-8) + (7)(3y) + (7)(-8)$

$= 15xy - 40x + 21y - 56$

#4. $(5m - \frac{2}{5}n)^2$

$= (5m - \frac{2}{5}n)(5m - \frac{2}{5}n)$

$= (5m)(5m) + (5m)(-\frac{2}{5}n) + (-\frac{2}{5}n)(5m) + (-\frac{2}{5}n)(-\frac{2}{5}n)$

$= 25m^2 - 2mn - 2mn + \frac{4}{25}n^2$

$= 25m^2 - 4mn + \frac{4}{25}n^2$

#5. $(2x-1)(3x^5 - 2x^3 + x^2 - 2x + 3)$

$= (2x)(3x^5) + (2x)(-2x^3) + (2x)(x^2) + (2x)(-2x) + (2x)(3)$

$+ (-1)(3x^5) + (-1)(-2x^3) + (-1)(x^2) + (-1)(-2x) + (-1)(3)$

$= 6x^6 - 4x^4 + 2x^3 - 4x^2 + 6x - 3x^5 + 2x^3 - x^2 + 2x - 3$

$= 6x^6 - 3x^5 - 4x^4 + 4x^3 - 5x^2 + 8x - 3$

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#6.

$$(p-3)^3 = (p-3)(p-3)(p-3)$$
$$= [(p)(p) + (p)(-3) + (p)(-3) + (-3)(-3)](p-3)$$
$$= [p^2 - 3p - 3p + 9](p-3)$$
$$= [p^2 - 6p + 9](p-3)$$
$$= (p^2)(p) + (p^2)(-3) + (-6p)(p) + (-6p)(-3) + (9)(p) + (9)(-3)$$
$$= p^3 - 3p^2 - 6p^2 + 18p + 9p - 27$$
$$= p^3 - 9p^2 + 27p - 27$$

#7.

$$\frac{-8h^6 + 36h^4 - 4h^2 - 26h + 7}{-4h^2} = \frac{-8h^6}{-4h^2} + \frac{36h^4}{-4h^2} - \frac{4h^2}{-4h^2} - \frac{26h}{-4h^2} + \frac{7}{-4h^2}$$
$$= 2h^4 - 9h^2 + 1 - \frac{13}{2h} - \frac{7}{4h^2}$$

or

$$= 2h^4 - 9h^2 + 1 - \frac{13}{2}h^{-1} - \frac{7}{4}h^{-2}$$

#8.

$$\frac{9w^2 + 6w + 10}{3w - 2} = 3w + 4 + \frac{18}{3w - 2}$$

$$\begin{array}{r|l} & 3w + 4 \\ \hline 3w - 2 & 9w^2 + 6w + 10 \\ & -(9w^2 - 6w) \\ \hline & 12w + 10 \\ & -(12w - 8) \\ \hline & 18 \end{array}$$

① $3w(3w) = 9w^2$

② $3w(3w - 2) = 9w^2 - 6w$

① $3w(4) = 12w$

② $4(3w - 2) = 12w - 8$

check:

$(3w-2)(3w+4) + 18 = 9w^2 + 6w + 10$

$9w^2 + 12w - 6w - 8 + 18 = 9w^2 + 6w + 10$

$9w^2 + 6w + 10 = 9w^2 + 6w + 10$ ✓ TRUE!

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#9. $\dfrac{18-11t^2+9t+12t^3}{4t+3} = \dfrac{12t^3-11t^2+9t+18}{4t+3}$

$= 3t^2-5t+6$

$3t^2-5t+6$

$4t+3 \;\overline{)\;12t^3-11t^2+9t+18}$

$-(12t^3+9t^2)$

$-20t^2+9t$

$-(-20t^2-15t)$

$24t+18$

$-(24t+18)$

0

① $4t(3t^2) = 12t^3$

② $3t^2(4t+3) = 12t^3+9t^2$

① $4t(-5t) = -20t^2$

② $-5t(4t+3) = -20t^2-15t$

① $4t(6) = 24t$

② $6(4t+3) = 24t+18$

check!

$(4t+3)(3t^2-5t+6)+0 = 12t^3-11t^2+9t+18$

$12t^3-20t^2+24t+9t^2-15t+18 = 12t^3-11t^2+9t+18$

$12t^3-11t^2+9t+18 = 12t^3-11t^2+9t+18$ ✓ TRUE!

#11. $19p^2y-38p^2y^3$

$= (19p^2y)(1)-(19p^2y)(2y^2)$

$= 19p^2y(1-2y^2)$

#12. $3r(5x-1)+7(5x-1)$

$= (5x-1)(3r+7)$

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#10. $\frac{y^3-1}{y-1} = \frac{y^3+0y^2+0y-1}{y-1}$

$= y^2+y+1$

$$\begin{array}{r|l} & y^2+y+1 \\ \hline y-1 & y^3+0y^2+0y-1 \\ & -(y^3-y^2) \\ & \hline & y^2+0y \\ & -(y^2-y) \\ & \hline & y-1 \\ & -(y-1) \\ & \hline & 0 \end{array}$$

① $y(y^2) = y^3$
② $y^2(y-1) = y^3-y^2$

① $y(y) = y^2$
② $y(y-1) = y^2-y$

① $y(1) = y$
② $1(y-1) = y-1$

check.

$(y-1)(y^2+y+1)+0 = y^3-1$

$y^3+y^2+y-y^2-y-1 = y^3-1$

$y^3-1 = y^3-1$ ✓ TRUE!

#13. $5m^2+15mp-2mp-6p^2$

$= (5m^2+15mp)+(-2mp-6p^2)$

$= 5m(m+3p)-2p(m+3p)$

$= (m+3p)(5m-2p)$

#14. q^2-q-42

$= (q-7)(q+6)$

42
1, 42
2, 21
3, 14
6, 7

check

$(q-7)(q+6)$

$= q^2+6q-7q-42$

$= q^2-q-42$ ✓

TRUE!

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#15. $3t^3 + 27t^2 + 24t$

$= 3t(t^2 + 9t + 8)$

$= 3t(t + 8)(t + 1)$

8
1, 8
2, 4

#16. $2y^3 + 3y^2z^2 - 54yz^3$

$= zy(y^2 + 3yz - 54z^2)$

$= zy(y + 9z)(y - 6z)$

54
1, 54
2, 27
3, 18
6, 9

#17. $(3m-n)k^2 - 13(3m-n)k + 40(3m-n)$

$= (3m-n)(k^2 - 13k + 40)$

$= (3m-n)(k - 8)(k - 5)$

40
1, 40
2, 20
4, 10
5, 8

#18. $4r^2 + 3r - 10$

$= (4r - 5)(r + 2)$

4	10
1, 4	1, 10
2, 2	2, 5

F: $4r^2$ + O: $8r$ − I: $5r$ − L: 10

$4r^2 + 3r - 10$

#19. $10x^2 + 11x - 6$

$= (2x + 3)(5x - 2)$

10	6
1, 10	1, 6
2, 5	2, 3

F: $10x^2$ − O: $4x$ + I: $15x$ − L: 6

$10x^2 + 11x - 6$

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#20, $12k^3q^4 - 4k^2q^5 - kq^6$

$= kq^4(12k^2 - 4kq - q^2)$

$= kq^4(2k - q)(6k + q)$

$12k^2$	$+0$	$2kq$	
	$-$	$6kq$	$-q^2$

$12k^2 - 4kq - q^2$

12
1, 12
2, 6
3, 4

#21, $-18k^3 - 48k^2 + 66k$

$= -6k(3k^2 + 8k - 11)$

$= -6k(3k + 11)(k - 1)$

$3k^2$	-0	$3k$	
	$+$	$11k$	-11

$3k^2 + 8k - 11$

3	11
1, 3	1, 11

#22, $32a^2 - 8$

$= 8(4a^2 - 1)$

$= 8(2a - 1)(2a + 1)$

$4a^2$	$+0$	$2a$	
	$-$	$2a$	-1

$4a^2 + 0a - 1$

4	1
1, 4	1, 1
2, 2	

#23, $81w^2 + 16$ ← "Sum of squares"

is prime

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#24. $y^4 - 16$

$= (y^2+4)(y^2-4)$

$= (y^2+4)(y+2)(y-2)$

$16 = 4^2$

$4 = 2^2$

#25. $m^2 + \frac{2}{3}m + \frac{1}{9}$

$= (m + \frac{1}{3})(m + \frac{1}{3})$

F $[m^2]$ + O $[\frac{1}{3}m]$ + I $[\frac{1}{3}m]$ + L $[\frac{1}{9}]$

$m^2 + \frac{2}{3}m + \frac{1}{9}$

$= (m + \frac{1}{3})^2$

$\frac{1}{9} = \frac{1}{3} \cdot \frac{1}{3}$

$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

#26. $-18x^2 - 48xy - 32y^2$

$= -2(9x^2 + 24xy + 16y^2)$

$= -2(3x + 4y)(3x + 4y)$

F $[9x^2]$ + O $[12xy]$ + I $[12xy]$ + L $[16y^2]$

$9x^2 + 24xy + 16y^2$

$= -2(3x + 4y)^2$

9	
1	9
3	3

16	
1	16
2	8
4	4

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#27. $125t^3 + 8s^3$

$= (5t)^3 + (2s)^3$

$= (5t + 2s)[(5t)^2 - (5t)(2s) + (2s)^2]$

$= (5t+2s)(25t^2 - 10ts + 4s^2)$

#28. $729a^6 + 343k^3$

$= (9a^2)^3 + (7k)^3$

$= (9a^2 + 7k)[(9a^2)^2 - (9a^2)(7k) + (7k)^2]$

$= (9a^2+7k)[81a^4 - 63a^2k + 49k^2]$

#29. $4b(2b+3) = 36$

$8b^2 + 12b = 36$

$-36 + 8b^2 + 12b = -36 + 36$

$8b^2 + 12b - 36 = 0$

$4(2b^2 + 3b - 9) = 0$

$4(2b - 3)(b + 3) = 0$

2	9
1, 2	1, 9
	3, 3

Either

$4 = 0$, $2b - 3 = 0$, or $b + 3 = 0$

False

$3 + 2b - 3 = 3 + 0$

$2b = 3$

$\frac{1}{2} \cdot 2b = \frac{1}{2} \cdot 3$

$b = \frac{3}{2}$

$-3 + b + 3 = -3 + 0$

$b = -3$

Check:

$4\left(\frac{3}{2}\right)\left[2\left(\frac{3}{2}\right)+3\right] = 36$

$2 \cdot 3[3 + 3] = 36$

$6 \cdot [6] = 36$

$36 = 36$, TRUE!

$4(-3)[2(-3)+3] = 36$

$-12[-6+3] = 36$

$-12[-3] = 36$

$36 = 36$, TRUE!

Solution set is $\{\frac{3}{2}, -3\}$.

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#30.

$2k(k+3) = (3k+1)(k+3)$

$2k^2+6k = 3k^2+9k+k+3$

$2k^2+6k = 3k^2+10k+3$

$-(2k^2+6k)+2k^2+6k = (-2k^2+6k)+3k^2+10k+3$

$0 = k^2+4k+3$

$0 = (k+3)(k+1)$

3
1, 3

either

$k+3=0$, or $k+1=0$

$-3+k+3=-3+0$ | $-1+k+1=-1+0$

$k=-3$ | $k=-1$

Solution set is $\{-3,-1\}$

check:

$2(-3)[(-3)+3] = [3(-3)+1][(-3)+3]$

$-6[0] = [-9+1][0]$

$0=0$, TRUE!

$2(-1)[(-1)+3] = [3(-1)+1][(-1)+3]$

$-2[2] = [-3+1][2]$

$-4 = (-2)(2)$

$-4=-4$, TRUE!

#31.

$y^3 = 6y^2-8y$

$-6y^2+8y \quad -6y^2+8y$

$y^3-6y^2+8y = 0$

$y(y^2-6y+8)=0$

$y(y-2)(y-4)=0$

8
1, 8
2, 4

either

$y=0$ | $y-2=0$, or $y-4=0$

$2+y-2=2+0$ | $4+y-4=4+0$

$y=2$ | $y=4$

Solution set is $\{0,2,4\}$.

check

$(0)^3 = 6(0)^2-8(0)$

$0=0-0$

$0=0$, TRUE!

$(2)^3 = 6(2)^2-8(2)$

$8 = 6\cdot 4-16$

$8 = 24-16$

$8=8$, TRUE!

$(4)^3 = 6(4)^2-8(4)$

$64 = 6\cdot 16-32$

$64 = 96-32$

$64=64$, TRUE!

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#32.

Area = L · W

let x = width of audio-cassette box

width = x cm

length = (x + 4) cm

Area = 77 cm²

$77 = (x+4)(x)$

$77 = x^2 + 4x$

$-77 + 77 = -77 + x^2 + 4x$

$0 = x^2 + 4x - 77$

$0 = (x+11)(x-7)$

77
1, 77
7, 11

Either

$x + 11 = 0$, or $x - 7 = 0$

$-11 + x + 11 = -11 + 0$ | $7 + x - 7 = 7 + 0$

$x = -11$ | $x = 7$

Negative width?? | $x + 4 = 11$

Failed

check!

$77 = [(7)+4](7)$

$77 = [11](7)$

$77 = 77$, ✓

true

ANS: The width of the box is 7cm and the length is 11cm.

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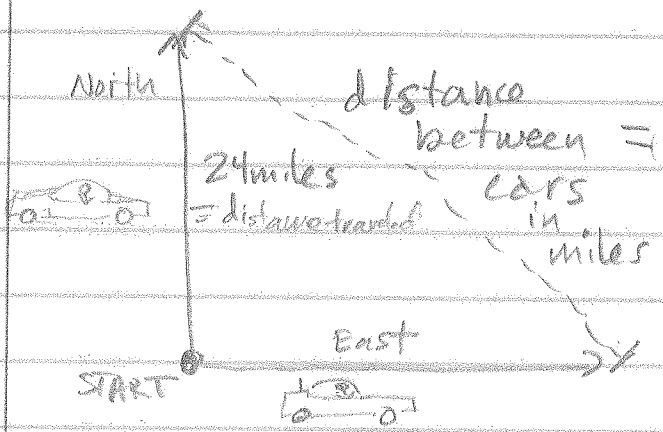
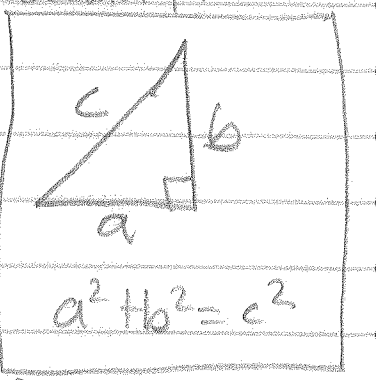
#33



Geometry Fact



Pythagorean Theorem

let x = distance traveled by eastbound car in miles

$(24)^2 + (x)^2 = (4+3x)^2$

$576 + x^2 = (4+3x)(4+3x)$

$576 + x^2 = 16 + 12x + 12x + 9x^2$

$576 + x^2 = 9x^2 + 24x + 16$

$-576 + x^2 + 576 + x^2 = -576 - x^2 + 9x^2 + 24x + 16$

$0 = 8x^2 + 24x - 560$

$0 = 8(x^2 + 3x - 70)$

$0 = 8(x-7)(x+10)$

70
1, 70
7, 10

Either

$8 = 0$, False | $x - 7 = 0$ | $+7 + x - 7 = +7 + 0$ | $x = 7$ | or $x + 10 = 0$ | $-10 + x + 10 = -10 + 0$ | $x = -10$ | False Negative Distance??

$4 + 3x = 4 + 3(7)$

$= 4 + 21$

$= 25$

check

$(24)^2 + (7)^2 = [4+3(7)]^2$

$576 + 49 = [4+21]^2$

$625 = (25)^2$

$625 = 625$ ✓

TRUE!

ANS: The distance between the cars would be 25 miles.

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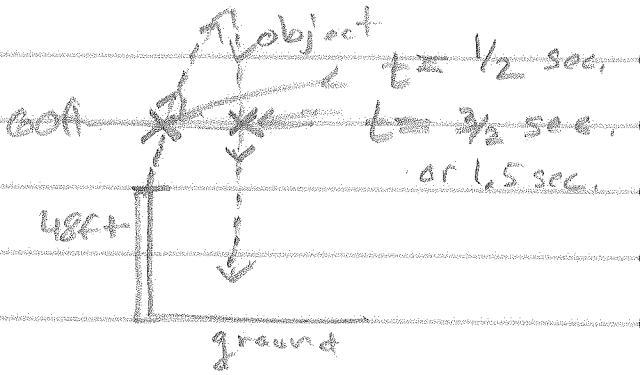
#34.

$h = -16t^2 + 32t + 48$

$h = 60$ find t:



$60 = -16t^2 + 32t + 48$

$-60 + 60 = -60 + (-16t^2 + 32t + 48)$

$0 = -16t^2 + 32t - 12$

$0 = -4(4t^2 - 8t + 3)$

$0 = -4(2t - 1)(2t - 3)$

$\frac{4}{1,4}$	$\frac{3}{1,3}$
$2, 2$	

Either

$-4 = 0$, $2t - 1 = 0$, or $2t - 3 = 0$

False!	$1 + 2t - 1 = 1 + 0$	$3 + 2t - 3 = 3 + 0$
	$2t = 1$	$2t = 3$
	$\frac{1}{2} \cdot 2t = \frac{1}{2} \cdot 1$	$\frac{1}{2} \cdot 2t = \frac{1}{2} \cdot 3$
	$t = \frac{1}{2}$	$t = \frac{3}{2}$

ANS: After $\frac{1}{2}$ second, the object is at a height of 60 feet, and after 1.5 seconds, the object is also at a height of 60 feet.